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Spectral questions of totally transcendental theory of finite rank

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synopsis

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Relevance of the topic. In the last decade it is rapidly developing a new field of mathematical logic - model theory. In the model theory there are two main topics in last two decades: spectral questions and classification of the spectral theory of functions.

In 1954 the hypothesis was stated by Dos' [1] : if the full theory T has only one model (exactly till isomorphism) in some uncountable cardinality, then each of other uncountable cardinalities also have only one theory of model.[1]

Equity of hypothesis of Dos' was proved by M.Morly in his works in 1965. [2] This work was very important for further research of full theries, which based on a new concept - concept of rank type. Theories, whose every type have rank, are called to be total trancendent. M.Morly has proved that categorical theories are total trancendent, and the total trancendence is equivalent to w - stability.[2]

Using a modification of the Morley rank, S.Shelakh [3] proved the validity of the Dos' hypothesis for odd languages. Herewith, there were allocated new classes of theory: superstabilized and stabilized. Countable theory is called to be superstabilized, if it is stabilized for any $\lambda \geq 2^w$. T is stabilized, is there exists $\lambda \geq \omega$, such than $T - \lambda$ - stabilized.

Let T is countable complete theorem of language α. Theorem of the spectrum function T is called by the next function :
I(T,α): $Or \to Card$
$\forall\alpha \in Or', I(T,\alpha) = |\{\frac{M}{\sim}/M \models T. \mid M \mid= \omega_\alpha.\}|$ where Or , $Card$-classes of ordinals and cardinals.

In 1971 A.Lachlan and D.Boldwin [4],by using strongly minimalize formulas and introducing by Marsh,they have proved next theory. If T is even, ω_1- categorical theory ,then
I(T,0)$\in \{1,\omega\}$.

[1]Such theories are called to be categorical

[2]The theory T is called to be λ stabilized ($\lambda \geq \omega$), if for any set A and any model M of theory T from $A \subset M, |A| = \lambda$ implies, that $|S_1(Th(A)) = \lambda$, where $S_1(Th(A))$ is the set of full 1 - types of language $L(A)$, joint with $Th(A)$

By summarizing dimension technique for strongly minimal formula,Lachlan show [6] that if T is superstable and $I(T,0) > 0$,then $I(T,0 \geq \omega)$

In 1971 E.Palutin [7],by the using of method minimalizing of set,he proved that if quasivariety categorically in the countable power,then it is categorically in the all non countable powers.

In 1970 ,Shelakh [8],then D.Rosenthal [9], proved that if T is totally transcendantal and $\omega_1$not categoric,then I(T,α) $\geq \mid \alpha + 1 \mid$.
In 1971[10] S.Shelah has proved that not stable theory of non countable powers have maximum number of nonisomorphic model pairs.In 1974 Shelokh[11] has proved next theorem:

Theorem 1 $\forall\alpha \geq 1$ and I(T,α) $= 2^{\omega_\alpha}$.

By this way unexplored spectral funcions totally transcendental theorems and superstable theorems. In 1977 O.Belegradeck[12] introduced the concept of whole categoric theorems and described spector of this theorie.A.Lachlan [13] by learning categoric theorem of ω,he has proved that superstable theorem of ω is totally transcendental.In 1975 A.Lachlan has answered question of D.Forresta and showed that even complete theorem which is finite,and not equal to one ,is by the pair of non isomorphic model and with one odd powers ω-categoric.We used technics of strongly minimalize formula by the proof of this fact .
In 1976 [15] A.Lachlan has introduced definable dimensionality theorem and proved totally transcendental theory rank is 2(r(x=x)=2), the power of I has definible dimension and by the help of this result ,he has described spectral function theorem rank is 2 with a power I.In 1975 T.Mustaphin[16] introduced strong types and strong basis theorem,and described spectral function theory with strong basis.In 1978 A.Lachlan [17] researched torally transcendental theorem.He has proved the next theorem.

Theorem 2 Let T is totally transcendental theory in a even langualge α.Then one of the next possibilty is true for the spectrum function.

(1) $I(T,0) \in 1,\omega$; $\forall\alpha \geq 1$. $I(T,\alpha)=1$
(2) $\forall \geq 0$ $I(T,\alpha) \leq max(\mid \alpha \mid, \omega)$;$\forall\alpha \geq \omega (I(T,\alpha)=\mid \alpha \mid)$
(3) $\forall\alpha \geq 1$ $I(T,\alpha)=\mid \alpha + 1 \mid^{\omega}$
(4) $\forall\alpha \geq 1$ $I(T,\alpha) \geq \omega^{\mid\alpha\mid}$

By disscussing possibilty (4) A.Lachlan expressed hypothesis:there exists totally transcendental theorem only for the next spectrum function $\forall\alpha \geq$1

a) I(T,α)=$\omega^{|\alpha|}$

b)I(T,α) $= 2^{\omega_\alpha}$

c)I(T,α) $= max(2^\omega * \omega^{|\alpha|})$ In 1979 Polyuton proved the next theory.[18]

Theorem 3 If K is complete variety ,then spectrum I(T(K),α) from K class defined as the on of the next :

(1)I(T(K),α)=1

(2)I(T(K),0)$\in \{1, \omega\}$ $\forall\alpha \geq$*1 I(T(K),α)=1*

(3) I(T(K),α) $= 2^{\omega_\alpha}$

If A$\subset$ M,then $S_n(A)$means that all complete n-type of set in A. Type $p \in S_n(\emptyset)$is called superstable if:

$\forall\alpha > 2^\omega$

$(\mid A \mid) \leq \lambda) \longrightarrow\mid \{q \in S_n(A) : p \subset q\} \mid\leq \lambda)$

In 1979 Mustaphin has proved two theorems.

Theorem 4 If complete even theorem has at least one secondary superstable type from $S_{n<m}S_n(\emptyset)$,then I(T,0)$\geq \omega$

Theorem 5 If stable theorem has two cardinal formulas,then:

$\forall\alpha \geq$*1 I(T,α)* $\geq\mid \alpha + 1 \mid$

From two theorem of Mustaphin ,we can define the theorm of Lachlan about number of even models and theorem of Shelakh-Rosenthal(as we said before).By this way,the result of last ten years show that increasing of interest to this direction.

Purpose of work : Researching spectrum complete theorem function-main purpose of thesis.We have 4 steps for the achievement of goals:

1)Describing totally transcendental theoryem rank 2,which have definite dimension

2)Describing spectrum function of all totally transcendental theorem rank 2

3)Receint enough condition to the totally transcendental theorem from the all odd powers which has maximum number of non isomorphic models

4)Finding condition ,which imposed complete theorem,for the $\forall\omega$-theory of stability follows the totally transcendental.

Scientific novilty :All result of thesis is new.In the thesis described general method of calculation number of models with the help of limitation theory on the formula:found new examples of totally transcendental theory with a different spectrum function;learning totally transcendental theory rank 2;found sufficent condition in order totally transcendental theory having maximum number of nonisomorphic pairs in a all odd powers.

Practical value :Result of thesis helps to find answers of specific algebric system.

Method of resarch :Method which has used authors to prove basic inves of thesis,directed to the result of rank of function,introduced by Morly,lachlan,Shelah and Mustaphin.

Approbation of work :By the topic of thesis published 6 work.All the results of thesis reported in a model theory seminar.

Workload :Thesis has written in a typescript form with 99 pages and 4 chapters.Bibliography contains 35 literary sources.

Content :In a 1 chapter ,we have given general form of thesis, preminilary reduction, which has some known facts about rank function in a different set. This chapter has proof of normalizing Lemma of Lacklan,Lemma of Shelakh about finite equivalent which has not full proof in the book.By the part,Lachlan[13] proved normalizng lemma for the rank Morly ω categories theorems. In a 3 has proof of normalizing lemma ψ rank and rank of complete theory of Morly.

Lemma 6 Let $\gamma(x,\bar{y})$ be a formula,p$(\bar{y})$-type which for any $\bar{b}$ realizing p.$\gamma(x,\bar{b})$ has ψ rank.Then exist $\gamma'(x,\bar{y})$,which for all $\bar{b}_0,\bar{b}_1$, realizing type p ,if formulas $\gamma(x,\bar{b}_c),\gamma(x,\bar{b}_0)\wedge\gamma(x,\bar{b}_1)$ which has same ψ-rank and ψ-powers,then $\models \forall x[\gamma'(x,\bar{b}_0)] \longleftrightarrow \gamma'(x,\bar{b}_1)$ and $\gamma'(x,\bar{b}_0),\gamma'(x,\bar{b}_1)$, $\gamma'(x,\bar{b}_0)\wedge\gamma'(x,\bar{b}_1)$ has same ψ-rank and same ψ powers. Then ,for any $\bar{b}'$ realizing type p,$\gamma'(x,\bar{b}_1)$ positive eqiuvalent boolean combination formulas $\gamma(x,\bar{b})$, where $\bar{b}$ realizing p.

We can see that general principle normalizing which we learned from Mustaphin in 1976.Obviously,R.Boot [20],complete theory can not have equally non isomorphic even models.R.Boot said hypothesis :for complete even theory T

$I(T,0) \in \{n/i \leq n < \omega\}_{n \neq z} \cup \{\omega, 2^\omega\}$

Of cource, this hypothesis makes sense at negation of the continuum hypothesis article.

Theorem 7 *(2.1.1) T Let T - complete theory of countable language and* $\mid \{D(M) \mid M \models T\} \mid > \omega$. *Then* $I(T,0) = 2^\omega$. *(Where* $D(M) = \{p \in S(T) \models p-implemented\ in\ M\}$*).*

In the proof of the theorem used Lemma 2.I.3. which is of independent interest as a modification of the compactness theorem A.I.Maltseva.

Lemma 2.1.3. Let given two countable sequences of non-principal types are $p_1, p_2, ..., p_m, ...; q_1, ..., q_m, ...m < \omega, p_m, q_m \in S(T)$. If for any $n < m$ there is a model M_n theory T, wherein all $p_m(m < n)$. implemented, a $q_m(m < \omega lowered.$

In the study spectral functions are important role play limiting theories of formula. In particular, for counting the number of models totally transcendental theory of rank 2,in Chapter 3 we used.

Theorem 8 *(2.2.1)* Let T be the total quasitotally transcendental theory of countable language L.Then for any infinite formula $\phi(x, \bar{a})$ (i.e. having an infinite number of solutions in models) any model N' theory $T' = T \wedge \phi(x, \bar{a})$ it can be reduced to models N theory T, where the region decision formula is a basic set of models N'.

Theorems 2.1.1 and 2.2.1 obtained by the author in a joint work with B.Omarov and included in the text of the thesis with the approval of the second author.

Let M model, $\phi(x, (\bar{y}))$ formula language L,$l((\bar{y})) = n$. Define model $M_{\phi(x,(\bar{y}))}$, language $L \cup R((\bar{y}), (\bar{z}) = L_\phi$ in the following way: on M^n we define an equivalence relation on

$\phi(x, (\bar{y})); (\bar{a}), (\bar{b}) \in M^n[\bar{a} \sim \phi(x, (\bar{y})), \bar{b} \Leftrightarrow \quad M \vdash \forall x[\phi(x, (\bar{a}) \leftrightarrow \phi(x, (\bar{b}))]]$

Theorem 9 *(2.2.3)* For all models M, N language L for any formulas $\phi(x, \bar{y})$.

$[M \equiv LN \Leftrightarrow M_\phi \equiv L_\phi N_\phi]$

This theorem is used to obtain sufficient conditions on the spectral function in Chapter IV.

We have already mentioned the hypothesis of Lachlan spectral functions totally transcendental theories.§3 Chapter II is devoted to the construction of examples to answer this question Proved.

Theorem 10 *(2.3.2)*

a) Let T complete theory of language L. $I(T,\alpha)$-function of spectar theory. Then there is a complete theory T^{ε} language L_{ε}, that

$I(T^{\varepsilon},\alpha) = min(2^{\omega_\alpha}, |\alpha+\omega|^{\cup_{\beta\leq\alpha} I(T,\beta)})$

b) If there are theories $T_1, T_2, ..., T_n, ...n < \omega$ with the following range of functions $\forall n I(T_n,\alpha) = min(2^{\omega_\alpha}, \delta(\lambda,\beta_n)), \beta_n < \beta, \cup(n < m)\beta_n = \beta(\delta(\lambda, k+1) = 2^{\delta(\lambda,k)})$ there is a complete theory T^p, that $I(T^p,\alpha) = min(2^{\omega_\alpha}, \delta(\lambda,\beta))$.

c) If $T, T_n n < \omega$ - stable, the $T^p : T^{\varepsilon}$ - stable.

Consequence(2.3.4) For any countable ordinal $\beta < \omega_1$ there are totally transcendental theory T_1, T_2 with the following range of functions:

a) $I(T_1,\alpha) = min(2^{\omega_\alpha}), \delta(|\alpha+1|,\beta))$

b) $I(T_2,\alpha) = min(2^{\omega_\alpha}, \delta(|\alpha+1|^{\omega}), \beta)$

T. Mustafin in Union Conference on the following hypothesis was formulated in mathematical logic.

Let T complete, countable but ω_1-categorical theory. If any formula theory T 1-cardinally there is an uncountable cardinal λ such that

$\forall\alpha \geq 1 I(T,\alpha) \leq \lambda$

Next theorem answers that question.

Theorem 11 *(2.3.5)* Exsist complete theory T countable language L such that any formula theory T, 1-cardinally and theory T have next function range.

$\forall\alpha \geq 0 I(T,\alpha) = min(2^{\omega_\alpha}, |\alpha+\omega|^{2^{\omega}})$

As mentioned above in [15] Lachlan introduced the concept of the theory of having a well-defined dimension Taimanov was asked to investigate the author totally transcendental theory of rank 2, with this dimension.

Set A model M called independent, if for any formulas $\phi(x,\bar{y})$ and any a,$b_1,...,b_n \subseteq Aa \notin b_1,...,b_n, b_i \neq_j$ $[M \vdash \varphi(a,\bar{(b)}) follow r(\varphi(x,\bar{(b)}) = r(x=x)]$. There $r(\varphi(x,\bar{(b)})$. Model M it has dimension if all maximal independent sets have the same cardinality. The theory T has a well-defined dimension if all its models have dimension.

Let $r(\psi(x,\bar{(c)}) = \beta > 0, d(\psi(x,\bar{(c)}) = 1$ formula $\varphi(x,\bar{(y)}$ decomposes formula $\psi(x,\bar{(c)})$, if

1)$\forall\bar{b}(r(\varphi(x,\bar{b}) \wedge \psi(x,\bar{c}) < r(\psi(x,\bar{c})$

2)$\forall\alpha < \beta\forall n < \omega\exists\bar{b}_0,..,\bar{b}_{n-1}\forall i < j < nr(\varphi(x,\bar{b}_i))\wedge\wedge_{j\neq i}\neg\varphi(x,b_j)\wedge \psi(x,\bar{c}) \geq \alpha$ formula $\psi(x,\bar{c})$ called decomposable, if

1) $r(\psi(x,\bar{c})) = r(x=x)2)d(\psi(x,\bar{c})) = 1$ 3) exist decomposable formula.

Theory T is called irreducible, if not decomposable formula together with T. Theory T is weakly decomposable, if

1) there is exist decomposable formula 2) the conjunction of two formulas decomposable decomposable. Main result is Chapter 3 is theorems 3.1.4 and 3.1.8.

Theorem 12 *(3.1.4)(3.1.8).* Let T countable, totally transcendental theory of rank 2, the degree n. Then if T is irreducible, (weakly decomposable), all models have the dimension T.

In the description of the spectrum of the theory of rank 2, division 1, Lachlan made a mistake. In §2 describes all the spectral features of the theory of rank 2, is an example of the theory, clarifying the spectrum of Lachlan.

Theorem 13 *(3.2.1)* Let T - totally transcensental theory of rank 2. Then for function spectrum of the theory T truth is one of the following options: $\forall\alpha \geq 1$

1) $I(T,\alpha) = 1$

2) $I(T,\alpha) \leq max(|\alpha|,\omega)\forall\alpha \geq \omega I(T,\alpha) = |\alpha|$

3) $I(T,\alpha) = |\alpha+1|^{\omega}$

4) $I(T,\alpha) = \omega^{|\alpha|}$

5) $I(T,\alpha) = max(2^{\omega},\omega^{|\alpha|})$

6) $I(T,\alpha) = 2^{\omega_\alpha}$

7) $I(T,\alpha) = min(2^{\omega_\alpha},|\alpha+1|^{|\alpha+1|^{\omega}})$

U.Forrest [21] studied universal theory of using the apparatus developed in a complete theory, if the compact T, countable theory, the set of all universal proposals that appear on T is called a generalist theory and is denoted $T_\forall$. Locally, the joint against the $T_\forall$-set of quantifier-free formulas with one variable called $\forall$-type. A theory T is called $\forall-(\omega)$stable, if obogoshenii language countable set subject of constant power of the set $\forall$-types do not increase. Obviously if T is totally transcendental, then it $\forall-(\omega)$-stable. The question for any theory T of $\forall-(\omega)$-stability should be a total transcendence? [21, Problem12] Obviously, this is true for complete theories admitting quantifier elimination.

Proposition (4.1.2). Let T - countable complete theory. Then for T conditions 1) and 2) equivalent

1) a) T-model complete

b) $T_\forall$-has amalgamated property investments

2) T-admits elimination of quantifiers.

One of the answers to this question, and the question will be Forrest [21].

Theorem 14 Exsist countable complete, $\forall-(\omega)$ -stable

In §2 of IV we learn relations between formulas of total transcendental theories.In private , by using properties of RK order we get

Theorem 15 If between the types , defining φ_ρ-incomparable by RK-formula of total transcendental thery of finite rank exists strong connecting type,then

$\forall\alpha \geq 1 I(T,\alpha) = 2^{\omega_\alpha}$

In chapters of IV also we learn some properties of total transcendental theories of finite rank.

Lemma 16 Let T- even , theory of total transcendental rank. Then for any a, b

$[t(a;\oslash) = t(b;\oslash) \Longrightarrow (a \in cl(b) \Leftrightarrow b \in cl(a))]$

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